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Duality Symmetries and the Type II String Effective Action

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We discuss the duality symmetries of Type II string effective actions in nine, ten and eleven dimensions. As a by-product we give a covariant action underlying the ten-dimensional Type IIB supergravity theory. We apply duality symmetries to construct dyonic Type II string solutions in six dimensions and their reformulation as solutions of the ten-dimensional Type IIB theory in ten dimensions.

1. INTRODUCTION

An important application of duality symmetries in string theory [1] is their use as solution-generating transformations at the level of the low-energy string effective action. To be precise, given a solution to the (lowest order in α') string equations of motion with one or more isometries, the duality symmetries generate new “dual” solutions. For this purpose it is important to understand all the symmetries of the string effective action in the presence of isometries.

Symmetries in the presence of isometries can be understood most easily by dimensionally reducing the string effective action over the isometry directions [2]. In general the dimensionally reduced action has more symmetries than the original one. These extra symmetries come from the following two sources:

1. General linear transformations over the isometry directions. In case we dimensionally reduce n directions this leads to a $GL(n, \mathbb{R})$ symmetry.
2. Electric-Magnetic duality transformations. These extra symmetries may occur each time the D -dimensional reduced action contains a p -form antisymmetric tensor with $p + 1 = D/2$. Note that this symmetry is only realized on the equations of motion. In the context of string effective actions this symmetry enhancement happens for dimensions $D \leq 8$.

The complete group of duality symmetries in

the case of Type II theories is usually called U -duality [3]. It contains the target space duality (T -duality) and the strong/weak coupling duality (S -duality).

In the first part of this talk I will discuss the different kinds of Type II duality symmetries and their interrelationships for the simplest case of one isometry. We are thus naturally led to consider effective actions in nine and ten dimensions. Since the Type IIA effective action can be obtained by dimensional reduction of eleven-dimensional supergravity it is natural to consider eleven dimensions as well. Further motivations are provided by the fact that eleven-dimensional supergravity is related to the eleven-dimensional supermembrane [4] and to the ten-dimensional Type IIA superstring [5,6]. The discussion below will be at the classical level and we will be dealing with continuous duality symmetries. It is well-known that these duality symmetries get broken to discrete subgroups at the quantum level. A discussion of Type II duality symmetries in $D = 9, 10, 11$ from a more stringy point of view can be found in [7].

2. DUALITY SYMMETRIES IN $D = 9, 10, 11$

We will organize our discussion with increasing number of isometries and decreasing number of dimensions. This leads us to consider the following three cases:

- a. Eleven dimensions without isometries

- b. Eleven dimensions with one isometry. This part of the discussion is naturally tied up with a consideration of ten dimensions without isometries.
- c. Eleven dimensions with two isometries. This naturally leads us to consider the cases of ten dimensions with one isometry and nine dimensions without isometries as well.

Below we will use the results and conventions of [8,9]. Note that double hatted fields are eleven-dimensional, hatted fields are ten-dimensional and unhatted fields are nine-dimensional. Concerning the $SO^\dagger(1,1)$ scale transformations given below, it is useful to consider both scale transformations that leave the action invariant as well as scale transformations that scale the action with a given weight. The reason for this is that the two types of scale transformations are related via the process of simple dimensional reduction. Note that the presence of two scale transformations that scale the action implies a single scale transformation that leaves the action invariant.

2.1. No isometries

D=11 There is a single $SO^\dagger(1,1)_{\text{membrane}}$ symmetry that essentially counts the mass dimension of the different fields. The scale transformations (with continuous parameter α) of the eleven-dimensional fields and action are given by

$$\begin{aligned}\hat{g}' &= e^\alpha \hat{g}, & \hat{C}' &= e^{3\alpha/2} \hat{C}, \\ S^{(11)'} &= e^{9\alpha/2} S^{(11)}.\end{aligned}\quad (1)$$

We remind that the three-index gauge field \hat{C} is a pseudo-tensor that changes sign under improper eleven-dimensional g.c.t.'s.

2.2. One isometry

We take the isometry to be in the y direction.

D=11 In addition to the symmetries of the previous subsection, we have to consider the

subgroup of g.c.t.'s that preserve the condition that the fields do not depend on the coordinate y . This group is

$$GL(1, \mathbb{R}) = SO^\dagger(1,1)_y \times \mathbb{Z}_2^{(y)}. \quad (2)$$

Here $SO^\dagger(1,1)_y$ represents the proper coordinate transformation (with parameter β) $y \rightarrow e^\beta y$ and $\mathbb{Z}_2^{(y)}$ represents the improper coordinate transformation $y \rightarrow -y$.

D=10, Type IIA Taking into account that \hat{C} changes sign under the (now) internal $\mathbb{Z}_2^{(y)}$, the eleven-dimensional transformations become the group

$$SO^\dagger(1,1)_{\text{membrane}} \times SO^\dagger(1,1)_y \times \mathbb{Z}_2^{(y)} \quad (3)$$

of global symmetries of the equations of motion. The scale weights under the different $SO^\dagger(1,1)$'s and the sign changes under $\mathbb{Z}_2^{(y)}$ are given in Table 2, Appendix B of [9].

D=10, Type IIB The underlying Type IIB supergravity theory will be discussed in more detail in the next section. Here we only discuss the aspects related to duality. The Type IIB theory has a $SL(2, \mathbb{R})_{\text{IIB}}$ duality [3] which in the string frame acts on the fields as follows

$$\begin{aligned}\hat{J}'_{\hat{\mu}\hat{\nu}} &= |c\hat{\lambda} + d| \hat{J}_{\hat{\mu}\hat{\nu}}, \\ \hat{\lambda}' &= \frac{a\hat{\lambda} + b}{c\hat{\lambda} + d}, \\ \begin{pmatrix} \hat{B}_{\hat{\mu}\hat{\nu}}^{(1)'} \\ \hat{B}_{\hat{\mu}\hat{\nu}}^{(2)'} \end{pmatrix} &= \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} \hat{B}_{\hat{\mu}\hat{\nu}}^{(1)} \\ \hat{B}_{\hat{\mu}\hat{\nu}}^{(2)} \end{pmatrix},\end{aligned}\quad (4)$$

where $ad - bc = 1$ and $\hat{\lambda} = \hat{\ell} + ie^{-\hat{\phi}}$. We use the convention that $\hat{\phi}$ is the dilaton and $\hat{B}^{(1)}$ the Neveu/Schwarz-Neveu/Schwarz (NS-NS) axion. Other definitions of the dilaton and NS-NS axion are

possible which differ from $\hat{\varphi}$ and $\hat{B}^{(1)}$ by an $SL(2, \mathbb{R})_{\text{IIB}}$ rotation.

There are several interesting subgroups of $SL(2, \mathbb{R})_{\text{IIB}}$. One is the $(\mathbb{Z}_4)_{\text{IIB}}$ subgroup generated by the element with $b = 1, c = -1$ and $a = d = 0$. It leads to the following transformation rules for the fields:¹

$$\begin{aligned}\hat{\lambda}' &= -1/\hat{\lambda}, & \hat{j}'_{\hat{\mu}\hat{\nu}} &= |\hat{\lambda}| \hat{j}_{\hat{\mu}\hat{\nu}}, \\ \hat{B}^{(1)'}_{\hat{\mu}\hat{\nu}} &= -\hat{B}^{(2)}_{\hat{\mu}\hat{\nu}}, & \hat{B}^{(2)'}_{\hat{\mu}\hat{\nu}} &= \hat{B}^{(1)}_{\hat{\mu}\hat{\nu}}.\end{aligned}\quad (5)$$

This transformation inverts the string coupling constant for $\hat{\ell} = 0$, i.e. $(e^{\hat{\varphi}})' = 1/e^{\hat{\varphi}}$ when $\hat{\ell} = 0$ [8]. It therefore makes sense to identify $SL(2, \mathbb{R})_{\text{IIB}}$ as an S -duality group of the Type IIB superstring [3]. We note that the NS-NS axion $\hat{B}^{(1)}$ and the Ramond-Ramond (R-R) axion $\hat{B}^{(2)}$ are rather different in nature. On the one hand, the NS-NS axion is an elementary excitation whose coupling to the Type IIB superstring is described by a two-dimensional sigma model. On the other hand, the R-R axion has its origin in solitonic modes on the worldsheet and its coupling is not described by a similar sigma model. Note that under the $(\mathbb{Z}_4)_{\text{IIB}}$ transformation (5) the NS-NS axion is rotated into the R-R axion and *vice-versa*. In view of this the $(\mathbb{Z}_4)_{\text{IIB}}$ transformation, besides a “strong/weak coupling” side, also has an “electric-magnetic” side from the worldsheet point of view [8].

We remark that the $(\mathbb{Z}_4)_{\text{IIB}}$ strong/weak coupling duality (5) can be converted into a $(\mathbb{Z}_2)_{\text{IIB}}$ strong/weak coupling duality by multiplying it with a so-called $\tilde{\mathbb{Z}}_2^{(y)}$ transformation. The latter transformation can be obtained from the $\mathbb{Z}_2^{(y)}$ of the Type IIA superstring given in (3) by application of the Type II Buscher T -duality [10,11] whose ex-

plicit rules are given in [8]². One thus obtains the rules:

$$\begin{aligned}\hat{\lambda}' &= 1/\hat{\lambda}^*, & \hat{j}'_{\hat{\mu}\hat{\nu}} &= |\hat{\lambda}| \hat{j}_{\hat{\mu}\hat{\nu}}, \\ \hat{B}^{(1)'}_{\hat{\mu}\hat{\nu}} &= \hat{B}^{(2)}_{\hat{\mu}\hat{\nu}}, & \hat{B}^{(2)'}_{\hat{\mu}\hat{\nu}} &= \hat{B}^{(1)}_{\hat{\mu}\hat{\nu}}, \\ \hat{D}'_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} &= -\hat{D}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}},\end{aligned}\quad (6)$$

which, for $\hat{\ell} = 0$, again leads to $(e^{\hat{\varphi}})' = 1/e^{\hat{\varphi}}$. We conclude that one can either represent the strong/weak coupling duality of the Type IIB theory as a \mathbb{Z}_2 transformation (like in (6)) but then it is not part of $SL(2, \mathbb{R})_{\text{IIB}}$ or one represents it as a particular $SL(2, \mathbb{R})_{\text{IIB}}$ transformation (like in (5)) but then it is not a \mathbb{Z}_2 transformation.

Another interesting subgroup of the S -duality group $SL(2, \mathbb{R})_{\text{IIB}}$ is the scaling group $\widetilde{SO}^\dagger(1, 1)_y$. Again it can be obtained from the $SO^\dagger(1, 1)_y$ of Type IIA, given in (3), by application of a Type II Buscher T -duality. Similarly, one can also translate the $SO^\dagger(1, 1)_{\text{membrane}}$ of Type IIA to the Type IIB language. The results are given in Table 3, Appendix B of [9].

In summary, the total global symmetry group of the Type IIB equations of motion is given by

$$SL(2, \mathbb{R})_{\text{IIB}} \times \widetilde{SO}^\dagger(1, 1)_{\text{membrane}} \times \tilde{\mathbb{Z}}_2^{(y)}. \quad (7)$$

It is intriguing that these symmetries exactly combine into a $GL(2, \mathbb{R})_{\text{IIB}}$ group.

²Note that the Type II T duality rules of [8] are *only* valid in the presence of an isometry (in ten dimensions). Such an isometry has not yet been assumed in this subsection (it will be done in the next subsection). Nevertheless the $\tilde{\mathbb{Z}}_2^{(y)}$ rules of the Type IIB theory can be obtained from the $\mathbb{Z}_2^{(y)}$ rules of the Type IIA theory in this way. In hindsight, the fact that one obtains the right rules, means that the $\mathbb{Z}_2^{(y)}$ and $\tilde{\mathbb{Z}}_2^{(y)}$ rules, which in themselves do not require the presence of an isometry, have the special property that, assuming there is an isometry, the two symmetries become *identical* after dimensional reduction over the isometry direction.

¹The product of two \mathbb{Z}_4 transformations leads to the (trivial) \mathbb{Z}_2 transformation that leaves the coupling constant inert but changes the sign of both axions.

This is the symmetry group one would expect if the ten-dimensional type IIB theory could be obtained by dimensional reduction of a (hypothetical) twelve-dimensional theory with no global symmetries whatsoever.

2.3. Two isometries

We take the two isometries to be in the x and y direction.

D=11 Upon dimensional reduction to nine dimensions, the general linear transformations in xy space become the group

$$GL(2, \mathbb{R}) = SL(2, \mathbb{R}) \times SO^\dagger(1, 1)_{x+y} \times \mathbb{Z}_2^{(x)}.$$

The $SL(2, \mathbb{R})$ group contains the subgroup $SO^\dagger(1, 1)_{x-y}$ corresponding to the eleven-dimensional g.c.t. $x \rightarrow e^\alpha x, y \rightarrow e^{-\alpha} y$. The other factor $SO^\dagger(1, 1)_{x+y}$ corresponds to the eleven-dimensional g.c.t. $x \rightarrow e^\beta x, y \rightarrow e^\beta y$. Finally, $\mathbb{Z}_2^{(x)}$ corresponds to the improper g.c.t. $x \rightarrow -x$ or any other $SL(2, \mathbb{R})$ -rotated version of this.

D=10, Type IIA and B In the presence of an isometry (in ten dimensions), the Type IIA and Type IIB theories are related by Type II T duality [10,11,9]. There are other global symmetries which are not covariant from the ten-dimensional point of view. They become covariant when we rewrite the theories in nine-dimensional language and so we will discuss them below.

D=9, Type II In nine dimensions there is a single Type II theory whose global symmetry group is given by:

$$\begin{aligned} SL(2, \mathbb{R}) \times SO^\dagger(1, 1)_{x+y} \times \\ \times SO^\dagger(1, 1)_{\text{membrane}} \times \mathbb{Z}_2^{(x)}. \end{aligned} \quad (8)$$

The $SL(2, \mathbb{R})$ group is a symmetry of the action. From the Type IIB point of view it is the manifest $SL(2, \mathbb{R})_{\text{IIB}}$ symmetry of

the original theory. From the point of view of the Type IIA only the $SO^\dagger(1, 1)_y$ can be realized in the absence of an isometry (in ten dimensions). The weights of the different nine-dimensional fields are summarized in Table 4, Appendix B of [9].

We note that in the presence of an isometry in the x -direction the $(\mathbb{Z}_4)_{\text{IIB}}$ transformation (5) can be interpreted as the special linear transformation $x \rightarrow y, y \rightarrow -x$ in xy space. Similarly, in the presence of an isometry, the \mathbb{Z}_2 transformation (6) corresponds to the improper g.c.t. $x \rightarrow -y, y \rightarrow -x$.

3. $D = 10$ TYPE IIB SUPERGRAVITY

In the next section we will illustrate the application of duality as a solution-generating transformation through an example. Since this example involves the ten-dimensional Type IIB superstring, we collect in this section some useful data concerning the underlying Type IIB supergravity theory.

It is known [12] that the field equations of $D = 10$ type IIB supergravity [13] cannot be derived from a covariant action³. The only equation of motion that cannot be obtained from an action is that of the four-form gauge field \hat{D} . This equation of motion states that the field strength \hat{F} of \hat{D} is self-dual: $\hat{F} = *\hat{F}$. It follows that if one sets $\hat{F} = 0$ everywhere in the equations of motion, one should be able to obtain the resulting reduced set of equations from an action, by varying with respect to all fields but \hat{D} . This was done in Ref. [8].

One may go one step further and define a non-self-dual (NSD) theory underlying the Type IIB theory by the property that it has the same field content as the original theory but \hat{F} is not self-dual and, if one imposes self-duality in the field equations, one recovers the usual Type IIB equations of motion [15]. Since the self-duality condition has disappeared one can write down an action for this NSD theory. A useful property of the NSD action is that, when properly used, it leads

³A covariant action has been constructed in an extended superspace based on a particular free differential algebra [14].

to the correct action for the *dimensionally reduced* type IIB supergravity theory. We thus may avoid the dimensional reduction of the ten-dimensional type IIB field equations which is more complicated. We will make use of this property in the next section.

The action corresponding to the NSD theory has been given in [15]. In the string frame and with the notation and conventions of Ref. [8,9,15], the NSD action is given by ($i = 1, 2$):

$$\begin{aligned} \hat{S}_{\text{NSD-IIB}}^{\text{string}} = \frac{1}{2} \int d^{10}x \sqrt{-\hat{g}} \Big\{ & \\ e^{-2\hat{\varphi}} \left[-\hat{R}(\hat{g}) + 4(\partial\hat{\varphi})^2 - \frac{3}{4}(\hat{\mathcal{H}}^{(1)})^2 \right] & \\ -\frac{1}{2}(\partial\hat{\ell})^2 - \frac{3}{4} \left(\hat{\mathcal{H}}^{(2)} - \hat{\ell}\hat{\mathcal{H}}^{(1)} \right)^2 & \\ -\frac{5}{6}\hat{F}^2 - \frac{1}{96\sqrt{-\hat{g}}} \epsilon^{ij} \epsilon \hat{D} \hat{\mathcal{H}}^{(i)} \hat{\mathcal{H}}^{(j)} \Big\}. \end{aligned} \quad (9)$$

For the sake of completeness we list the definitions of the field strengths and gauge transformations for the type IIB fields $\{\hat{D}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}, \hat{g}_{\hat{\mu}\hat{\nu}}, \hat{\mathcal{B}}_{\hat{\mu}\hat{\nu}}^{(i)}, \hat{\ell}, \hat{\varphi}\}$:

$$\begin{aligned} \hat{\mathcal{H}}^{(i)} &= \partial \hat{\mathcal{B}}^{(i)}, \quad \delta \hat{\mathcal{B}}^{(i)} = \partial \hat{\Sigma}^{(i)}, \\ \hat{F} &= \partial \hat{D} + \frac{3}{4} \epsilon^{ij} \hat{\mathcal{B}}^{(i)} \partial \hat{\mathcal{B}}^{(j)}, \\ \delta \hat{D} &= \partial \hat{\rho} - \frac{3}{4} \epsilon^{ij} \partial \hat{\Sigma}^{(i)} \hat{\mathcal{B}}^{(j)}. \end{aligned} \quad (10)$$

The NSD theory defined by Eq. (9) has all the symmetries of the type IIB theory, given in the previous section (see eq. (7)) plus an additional global \mathbb{Z}_2 e-m duality of the \hat{D} field that interchanges \hat{F} and $*\hat{F}$.

The $SL(2, \mathbb{R})_{\text{IIB}}$ symmetries have already been given in eq. (4) of the previous section. To make the $SL(2, \mathbb{R})_{\text{IIB}}$ invariance of the action more manifest, it is useful to go to the Einstein frame because the Einstein metric is inert under them:

$$\begin{aligned} S_{\text{NSD-IIB}}^{\text{Einstein}} = \frac{1}{2} \int d^{10}x \sqrt{-\hat{g}} \Big\{ & \\ -\hat{R} & \\ +\frac{1}{4} \text{Tr} \left(\hat{\mu} \hat{\mathcal{M}} \partial^\mu \hat{\mathcal{M}}^{-1} \right) - \frac{3}{4} \hat{\mathcal{H}}^{(i)} \hat{\mathcal{M}}_{ij} \hat{\mathcal{H}}^{(j)} & \\ -\frac{5}{6} \hat{F}^2 - \frac{1}{96\sqrt{-\hat{g}}} \epsilon^{ij} \epsilon \hat{D} \hat{\mathcal{H}}^{(i)} \hat{\mathcal{H}}^{(j)} \Big\}, \end{aligned} \quad (11)$$

where $\hat{g}_{\hat{\mu}\hat{\nu}} = e^{-\frac{1}{2}\hat{\varphi}} \hat{g}_{\hat{\mu}\hat{\nu}}$ is the Einstein-frame metric. The matrix $\hat{\mathcal{M}}$ is the 2×2 matrix

$$\hat{\mathcal{M}} = \left(\hat{\mathcal{M}}_{ij} \right) = \frac{1}{\Im \lambda} \begin{pmatrix} |\hat{\lambda}|^2 & -\Re \hat{\lambda} \\ -\Re \hat{\lambda} & 1 \end{pmatrix}, \quad (12)$$

where $\hat{\lambda} = \hat{\ell} + ie^{-\hat{\varphi}}$ is a complex scalar that parametrizes $SL(2, \mathbb{R})_{\text{IIB}}$.

The action (11) is manifestly invariant under the $SL(2, \mathbb{R})_{\text{IIB}}$ transformations

$$\hat{\mathcal{H}}' = \Lambda \hat{\mathcal{H}}, \quad \hat{\mathcal{M}}' = (\Lambda^{-1})^T \hat{\mathcal{M}} \Lambda^{-1}. \quad (13)$$

Here $\hat{\mathcal{H}}$ denotes a 2-vector with $\hat{\mathcal{H}}^{(1)}$ as the upper component and $\hat{\mathcal{H}}^{(2)}$ as the lower component. If Λ is the $SL(2, \mathbb{R})_{\text{IIB}}$ matrix

$$\Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix}, \quad (14)$$

the transformation Eq. (13) of the matrix $\hat{\mathcal{M}}$ implies the usual transformation of the complex scalar $\hat{\lambda}$

$$\hat{\lambda}' = \frac{a\hat{\lambda} + b}{c\hat{\lambda} + d}. \quad (15)$$

Finally, we expect that a NSD type IIB theory including fermions can also be found. Of course, the full NSD type IIB action cannot be supersymmetric since the self-duality constraint has no fermionic partner. However, the supersymmetry should be recovered in the field equations of the Type IIB theory after the (super-) self-duality constraint is imposed⁴.

4. DYONIC STRINGS IN SIX DIMENSIONS

As an application of duality as a solution-generating transformation we will construct in this section dyonic string solutions in six dimensions⁵. The reformulation of these dyonic strings as solutions of the Type IIB theory in ten dimensions will be discussed in the next section. We only consider solutions to the source-free equations of

⁴We thank M. Green for a discussion on this point.

⁵A similar construction of dyonic membranes in eight dimensions has recently been given in [16].

motion. Without sources, we define a p -brane solution to be any solution with p translational spacelike isometries.

It is well-known that reducing $D = 10$ Type IIB supergravity over a 4-torus leads to the nonchiral $D = 6$ Type IIA supergravity theory constructed in [17]⁶. This theory has a noncompact $SO(5, 5)$ U -duality. For our purposes it is enough to consider a truncated version of the $D = 6$ Type IIA theory in which only a subgroup $SO(2, 2)$ of the full U -duality group is realized.

To be explicit, reducing the ten-dimensional Type IIB fields to six dimensions we consider the following ansatz for the fields (μ, ν, \dots are six-dimensional spacetime indices and m, n, \dots are the four internal directions):

$$\begin{aligned}\hat{j}_{\mu\nu} &= j_{\mu\nu}, & \hat{j}_{mn} &= -e^{G+\varphi/2}\delta_{mn}, \\ \hat{B}_{\mu\nu}^{(i)} &= B_{\mu\nu}^{(i)}, \\ \hat{D}_{\mu\nu\rho\sigma} &= D_{\mu\nu\rho\sigma}, & \hat{D}_{mnpq} &= D_{mnpq}, \\ \hat{\ell} &= \ell, & \hat{\varphi} &= \varphi.\end{aligned}\quad (16)$$

All other components are zero. Concerning the D field, we observe that both D_{mnpq} and $D_{\mu\nu\rho\sigma}$ (after dualization) lead in six dimensions to scalars $D = \epsilon^{mnpq}D_{mnpq}$ and \tilde{D} , respectively. Using a suitable normalization condition for \tilde{D} turns the self-duality constraint into $\tilde{D} = D$, which can now be substituted into the NSD action. It is therefore enough and consistent to only collect the D_{mnpq} terms in the dimensional reduction and to multiply these terms by a factor two.

The resulting reduced action is

$$\begin{aligned}S &= \frac{1}{2} \int d^6x \sqrt{-j} \left[e^{-2\varphi} \left[-R(j) + 4(\partial\varphi)^2 \right. \right. \\ &\quad - (\partial\tilde{G})^2 - \frac{3}{4}(\mathcal{H}^{(1)})^2 \left. \right] - \frac{1}{2}e^{2\tilde{G}}(\partial\ell)^2 \\ &\quad - \frac{3}{4}e^{2\tilde{G}}(\mathcal{H}^{(2)} - \ell\mathcal{H}^{(1)})^2 \\ &\quad \left. - \frac{1}{72}e^{-2\tilde{G}}(\partial D)^2 + \frac{1}{8}D\mathcal{H}^T\mathcal{L}^*\mathcal{H} \right],\end{aligned}\quad (17)$$

where $(*\mathcal{H})_{\mu\nu\rho} = \frac{1}{6\sqrt{-j}}\epsilon_{\mu\nu\rho\alpha\beta\gamma}\mathcal{H}^{\alpha\beta\gamma}$, $\tilde{G} \equiv G +$

⁶The reduction of $D = 10$ Type IIA supergravity over a 4-torus has recently been considered in [18].

$\varphi/2$, and where we have introduced the 2×2 matrix

$$\mathcal{L} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (18)$$

To make the symmetries manifest, it is convenient to go to the six-dimensional Einstein metric $g_{\mu\nu} = e^{-\varphi}j_{\mu\nu}$. We thus obtain

$$\begin{aligned}S &= \frac{1}{2} \int d^6x \sqrt{-g} \left[-R(g) \right. \\ &\quad + \frac{2\partial\lambda\partial\bar{\lambda}}{(\lambda - \bar{\lambda})^2} + \frac{2\partial\kappa\partial\bar{\kappa}}{(\kappa - \bar{\kappa})^2} \\ &\quad \left. - \kappa_2\mathcal{H}^T\mathcal{M}\mathcal{H} + \kappa_1\mathcal{H}^T\mathcal{L}^*\mathcal{H} \right].\end{aligned}\quad (19)$$

The complex scalars λ and κ are defined by

$$\begin{aligned}\kappa &= \kappa_1 + i\kappa_2 = \frac{1}{8}D + \frac{3}{4}ie^{2G}, \\ \lambda &= \lambda_1 + i\lambda_2 = \ell + ie^{-\varphi}.\end{aligned}\quad (20)$$

We see that there are two $SL(2, \mathbb{R})/U(1)$ scalar cosets in the action (19) and correspondingly there are two $SL(2, \mathbb{R})$ symmetries of the equations of motion. One of them is the original $SL(2, \mathbb{R})_{\text{IIB}}$ symmetry of the NSD type IIB action (9). Note that G and not \tilde{G} is $SL(2, \mathbb{R})_{\text{IIB}}$ invariant. The second is an electro-magnetic $SL(2, \mathbb{R})_{\text{EM}}$ duality that acts on the two-form potentials. The latter is only a symmetry of the equations of motion. It acts on κ and \mathcal{H} as follows:

$$\begin{aligned}\kappa' &= \frac{p\kappa + q}{r\kappa + s}, \\ \mathcal{H}'_{\mu\nu\rho} &= (r\kappa_1 + s)\mathcal{H}_{\mu\nu\rho} + r\kappa_2\mathcal{L}\hat{\mathcal{M}}^*\mathcal{H}_{\mu\nu\rho},\end{aligned}\quad (21)$$

with $ps - qr = 1$. A particularly interesting $(\mathbb{Z}_2)_{\text{EM}}$ subgroup of $SL(2, \mathbb{R})_{\text{EM}}$ is generated by the element with $q = -3/4, r = 4/3$ and $p = s = 0$. Note that the $SL(2, \mathbb{R})_{\text{EM}}$ transformations are similar in form to the S duality of the heterotic string compactified to four dimensions⁷, with vector fields replaced by two-form fields and with the axion/dilaton field replaced by κ .

⁷See e.g. the review of [19].

In summary, we have recovered a

$$SO(2, 2) \equiv SL(2, \mathbb{R})_{\text{EM}} \times SL(2, \mathbb{R})_{\text{IIB}} \quad (22)$$

noncompact symmetry of the six-dimensional equations of motion.

We are now in a position to construct six-dimensional dyonic string solutions. Our starting point is the following six-dimensional solitonic string solution in the string frame [20]:

$$1_{(6)\text{m}}^{(1)} \left\{ \begin{array}{l} ds^2 = (dx^0)^2 - (dx^1)^2 - e^{2\phi}(dx^a)^2, \\ \mathcal{H}_{abc}^{(1)} = \frac{2}{3}\epsilon_{abcd}\partial^d\varphi, \\ \varphi = \varphi(x^a), \quad \square e^{2\varphi} = 0, \end{array} \right.$$

where $\square = \delta^{ab}\partial_a\partial_b$. The superscript (1) indicates that the charge of the solution is carried by the NS-NS axion $\mathcal{B}^{(1)}$. We have parametrized the six-dimensional space by $x^\mu = (x^0, x^1, x^a)$, $a \in \{6, 7, 8, 9\}$.

We next apply to the $1_{(6)\text{m}}^{(1)}$ solution given above the most general $SL(2, \mathbb{R})_{\text{IIB}} \times SL(2, \mathbb{R})_{\text{EM}}$ transformation, with parameters a, b, c, d ($ad - bc = 1$) (see eq. (4)) and p, q, r, s ($ps - qr = 1$) (see eq. (21)), respectively. The result is given by

$$\begin{aligned} ds^2 &= A [(dx^0)^2 - (dx^1)^2 - e^{2C}(dx^a)^2], \\ \begin{pmatrix} \mathcal{H}^{(1)} \\ \mathcal{H}^{(2)} \end{pmatrix} &= \begin{pmatrix} dsH - \frac{3}{4}cre^{-2C} * H \\ bsH - \frac{3}{4}are^{-2C} * H \end{pmatrix}, \\ \ell &= \frac{bd + ace^{-2C}}{d^2 + c^2e^{-2C}}, \\ e^{-\varphi} &= \frac{e^{-C}}{d^2 + c^2e^{-2C}}, \\ D &= 8 \frac{qs + \frac{9}{16}pre^{-2C}}{s^2 + \frac{9}{16}r^2e^{-2C}}, \\ e^{2G} &= \frac{e^{-C}}{s^2 + \frac{9}{16}r^2e^{-2C}}, \end{aligned} \quad (23)$$

where A and H_{abc} are functions of C

$$\begin{aligned} A &= \sqrt{d^2 + c^2e^{-2C}} \sqrt{s^2 + \frac{9}{16}r^2e^{-2C}}, \\ H_{abc} &= \frac{2}{3}\epsilon_{abcd}\partial^d C, \end{aligned} \quad (24)$$

and C depends only on the x^a 's and satisfies $\square e^{2C} = 0$.

A characteristic feature of the above dyonic string solutions is that non-zero R-R fields are needed in order for the solution to carry electric as well as magnetic charge. Setting the R-R axion $\mathcal{B}^{(2)}$ and the other R-R fields ℓ and D equal to zero, leads to a purely electric or magnetic solution.

The family of dyonic string solutions (23) contains four purely electrically or magnetically charged string solutions as special cases⁸. First of all, for $a = d = p = s = 1, b = c = q = r = 0$ (unit transformation) we recover the original solution $1_{(6)\text{m}}^{(1)}$.

Secondly, one may perform the $(\mathbb{Z}_4)_{\text{IIB}}$ transformation (5) which corresponds to the case $c = p = s = 1, b = -1, a = d = q = r = 0$. When acting on the $1_{(6)\text{m}}^{(1)}$ solution, it leads to the electrically⁹ charged solution $1_{(6)\text{e}}^{(2)}$:

$$1_{(6)\text{e}}^{(2)} \left\{ \begin{array}{l} ds^2 = e^\varphi [(dx^0)^2 - (dx^1)^2] - e^{-\varphi}(dx^a)^2, \\ \mathcal{H}_{abc}^{(2)} = \frac{2}{3}\epsilon_{abcd}\partial^d\varphi, \quad e^{2G} = e^\varphi, \\ \varphi = \varphi(x^a), \quad \square e^{-2\varphi} = 0. \end{array} \right.$$

Thirdly, a $1_{(6)\text{e}}^{(1)}$ solution may be obtained by applying the $(\mathbb{Z}_4)_{\text{IIB}} \times (\mathbb{Z}_2)_{\text{EM}}$ transformation $c = 1, b = -1, q = -3/4, r = 4/3, a = d = p = s = 0$ on the $1_{(6)\text{m}}^{(1)}$ solution. It is given by [21]

$$1_{(6)\text{e}}^{(1)} \left\{ \begin{array}{l} ds^2 = e^{2\varphi} [(dx^0)^2 - (dx^1)^2] - (dx^a)^2, \\ \mathcal{B}_{01}^{(1)} = e^{2\varphi}, \\ \varphi = \varphi(x^a), \quad \square e^{-2\varphi} = 0. \end{array} \right.$$

⁸Strictly speaking there are four more solutions that can be obtained by acting with a $(\mathbb{Z}_4)_{\text{IIB}}^2$ transformation on the four solutions given below. These extra solutions only differ in the sign of the axions.

⁹We define the R-R charge of this solution to be of electric character because the string coupling constant e^φ is small ($e^{-2\varphi}$ is singular). According to this definition *all* solutions (both the ones that carry a NS-NS charge and the ones that carry a R-R charge) have the property that the dilaton corresponding to the electrically (magnetically) charged solutions satisfies $\square e^{-2\varphi} = 0$ ($\square e^{2\varphi} = 0$). In this way the electric and magnetic solutions are always connected via a strong/weak coupling duality.

Finally, by applying the $(\mathbb{Z}_2)_{\text{EM}}$ transformation $a = d = 1, q = -3/4, r = 4/3, b = c = p = s = 0$ on the $1_{(6)\text{m}}^{(1)}$ solution we obtain a second magnetically charged solution $1_{(6)\text{m}}^{(2)}$:

$$1_{(6)\text{m}}^{(2)} \left\{ \begin{array}{ll} ds^2 &= e^{-\varphi} [(dx^0)^2 - (dx^1)^2] - e^{\varphi} (dx^a)^2, \\ B_{01}^{(2)} &= -e^{-2\varphi}, \quad e^{2G} = e^{\varphi}, \\ \varphi &= \varphi(x^a), \quad \square e^{2\varphi} = 0. \end{array} \right.$$

It is interesting to compare the six-dimensional dyonic string solutions (23) with the six-dimensional dyonic string solution that was recently constructed in [22]. Our solution differs from that of [22] in the following two respects. First of all, the dyonic string of [22] contains no R-R fields whereas our solution does. For instance, our generic solution contains two axions while the one of [22] contains one. Secondly, the solution of [22] is a heterotic solution that breaks 3/4 of the spacetime supersymmetries. Our solution is a Type II solution that breaks 1/2 of the Type II spacetime supersymmetries. This is necessarily so because the solitonic string we started with has this property and we know that the $SL(2, \mathbb{R})_{\text{IIB}} \times SL(2, \mathbb{R})_{\text{EM}}$ transformation, viewed as a noncompact symmetry of six-dimensional supergravity, is consistent with the full set of type II supersymmetries.

5. STRING/FIVE-BRANES IN TEN DIMENSIONS

The six-dimensional dyonic string solutions constructed in the previous section can be reformulated as solutions of the equations of motion corresponding to the Type IIB theory in ten dimensions. This can be done in a straightforward manner by using the relation (16) between the six-dimensional and ten-dimensional fields. The explicit form of the ten-dimensional solutions can be found in [15] and will not be repeated here.

Sofar the discussion has been restricted to the source-free equations only. The situation with the source terms is less clear. In this context, see also [16,23,24]. A complicating feature is that our solutions are intrinsic Type II solutions, i.e. with a non-zero R-R axion. One cannot treat this field

as the NS-NS axion whose coupling to the superstring is described by a two-dimensional sigma model. For the solutions with $p = s = 1$ and $q = r = 0$, an alternative approach is to *define* the NS-NS axion to be the $SL(2, \mathbb{R})_{\text{IIB}}$ rotation of $\hat{B}^{(1)}$ such that the NS-NS axion carries all the charge and the R-R axion none.

Restricting ourselves for the moment to the four special solutions of the previous section, one may verify that, including the source terms, they correspond to the following string and fivebrane solutions of the ten-dimensional Type IIB theory [21,25,26,3]:

$$\begin{aligned} 1_{(6)\text{e}}^{(1)} &\rightarrow 1_{(10)\text{e}}^{(1)}, & 1_{(6)\text{m}}^{(1)} &\rightarrow 5_{(10)\text{m}}^{(1)}, \\ 1_{(6)\text{e}}^{(2)} &\rightarrow 5_{(10)\text{e}}^{(2)}, & 1_{(6)\text{m}}^{(2)} &\rightarrow 1_{(10)\text{m}}^{(2)}. \end{aligned} \quad (25)$$

In relating the ten-dimensional solutions to the six-dimensional solutions, the ten-dimensional strings are reduced over four *transverse* directions whereas the five-branes are reduced over four *world-volume* directions. We observe that some of the six-dimensional string solutions have a natural interpretation as a ten-dimensional string whereas others become five-branes in ten dimensions. It would be interesting to see whether the ten-dimensional solutions with $a = d = 1$ and $b = c = 0$ would provide for a natural interpolation between strings and five-branes in ten dimensions thereby giving further evidence for a ten-dimensional string/fivebrane duality of the type proposed in [27], like it has been suggested in [16] for the case of the conjectured $D = 11$ membrane/fivebrane duality [3,28].

Finally, a subset of the ten-dimensional solutions, corresponding to the case $p = s = 1$ and $q = r = 0$ has been considered recently from a more stringy point of view and shown to describe a whole $SL(2, \mathbb{Z})$ multiplet of Type IIB superstrings [23]. Note that two special members of this multiplet are the ten-dimensional $1_{(10)\text{e}}^{(1)}$ and $1_{(10)\text{m}}^{(2)}$ solutions defined above. These two solutions have been used in [29,30] to support evidence for the $D = 10$ string/string duality between the $SO(32)$ heterotic and Type I superstring proposed in [6].

6. DISCUSSION

The techniques we used in constructing the six-dimensional dyonic string solutions and their reformulation as ten-dimensional dyonic string/five-brane solutions can be applied to more general cases as well. To explain the basic idea¹⁰, consider a supergravity theory containing a metric $g_{\mu\nu}$, a dilaton φ and a $(p+1)$ -form gauge field $B_{\mu_1\cdots\mu_{p+1}}$ ¹¹. The Lagrangian for these fields takes a standard form, as in [20]. From the general analysis of [20] it follows that this theory has an elementary p -brane solution p_e and a solitonic $(D-p-4)$ -brane solution $(D-p-4)_m$. We next observe that the dual of a $(p+1)$ -form field is again a $(p+1)$ -form field in $2(p+2)$ spacetime dimensions. In order to allow for a \mathbb{Z}_2 duality transformation we therefore reinterpret the $(D-p-4)_m$ -solution in D dimensions as a p_m -solution in $2(p+2)$ dimensions via a dimensional reduction over $D-2p-4$ space-like *worldvolume* directions. Similarly, a dimensional reduction of the p_e -solution in D dimension over $D-2p-4$ spacelike *transverse* directions leads to a p_e -solution in $2(p+2)$ dimensions. Given the standard form of the Lagrangian in D dimensions, and assuming a simple ansatz for the D -dimensional fields that includes the D -dimensional p_e and $(D-p-4)_m$ solutions (as in (16)), one can, for all cases we have considered, show that the field equations corresponding to the dimensionally reduced theory in $2(p+2)$ dimensions are invariant under a \mathbb{Z}_2 duality transformation that maps the p_e and p_m solutions into each other. In summary, the special case of the p_e solution in D dimensions where there are $(D-2p-4)$ extra abelian isometries in the transverse directions can be viewed, via a \mathbb{Z}_2 duality transformation in $2(p+2)$ dimensions, as the purely “electrically” charged partner of the “magnetically” charged $(D-p-4)_m$ soliton solution in D dimensions.

It is instructive to consider a few examples of

the above general analysis. Consider for instance ten-dimensional type IIA supergravity. The theory contains a 1, 2- and 3-form gauge fields and therefore has the following solutions (see e.g. [31] or the table in [32]):

$$(0_e, 6_m), \quad (1_e, 5_m), \quad (2_e, 4_m). \quad (26)$$

Applying the above analysis for $D = 10$ and $p = 0, 1, 2$, respectively, we see that all the elementary solutions can be reinterpreted as purely “electrically” charged partners of the “magnetically” charged soliton solutions by a \mathbb{Z}_2 duality transformation in 4, 6 and 8 dimensions, respectively.

Next, we consider type IIB supergravity. It contains a complex 2-form and a self-dual 4-form gauge field. The complex 2-form gauge field leads to the following solutions (using the definition of “electric” and “magnetic” we adopted for the R-R axionic charges):

$$(1_e + 1_m, 5_e + 5_m). \quad (27)$$

In addition, the self-dual 4-form gauge field leads to the self-dual threebrane solution 3_{em} of [33,34]. By reducing to six dimensions we find that the $1_e + 1_m$ solutions can be considered as the electrically + magnetically charged partners of the $5_m + 5_e$ solutions. This is the case discussed in this talk.

The self-dual 3_{em} solution is unique in the sense that our general formulae given above lead to a \mathbb{Z}_2 -duality transformation in ten dimensions itself. However, since type IIB supergravity is already self-dual there is no such \mathbb{Z}_2 -transformation.

Finally, we consider the case of eleven-dimensional supergravity. There is only one 3-form in eleven dimensions which leads to an elementary membrane 2_e [35] and a solitonic five-brane 5_m [36]. By performing a \mathbb{Z}_2 duality transformation in 8 dimensions we find that the 2_e is the electrically charged partner of the 5_m solution.

The results of this work suggest that in all examples given above, the \mathbb{Z}_2 duality transformation can be extended to an $SL(2, \mathbb{R})$ transformation in a relatively simple way. The $SL(2, \mathbb{R})$ -transformations so obtained can then be applied

¹⁰The discussion below is similar to that of [31].

¹¹We only consider p -brane solutions where the charge is carried by $B_{\mu_1\cdots\mu_{p+1}}$. In particular, we do not consider p -brane solutions where the charge is carried by a vector component of the metric g .

to construct dyonic p -brane solutions in $2(p+2)$ dimensions and their reformulation as $p_e/(D-p-4)_m$ solutions in D dimensions. In fact, recently a further example has been constructed for the case of $p=2, D=11$ corresponding to dyonic membranes in 8 dimensions and their reformulation as membrane/five-brane solutions in eleven dimensions [16].

It would be interesting to construct further examples of dyonic p -brane solutions and to investigate their properties, like *e.g.* their singularity structure.

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